Adjacent vertex distinguishing edge coloring on complete split graphs and split-indifference graphs

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ABSTRACT

A proper edge coloring of a graph G is an assignment of colors to the edges of G so that the colors of any two adjacent edges are distinct. Given a graph G with a proper edge coloring, the set of colors of a vertex v is the set of colors assigned to the edges incident to v. Two vertices are distinguishable if their sets of colors are distinct. An adjacent vertex distinguishing (AVD) edge coloring of G is a proper edge coloring such that every two adjacent vertices are distinguishable. The minimum number of colors to an AVD edge coloring of a graph G is the AVD chromatic index. There are partial results on the AVD chromatic index for the classes of split-indifference graphs and complete split graphs. In this paper we present the AVD chromatic index for the remaining graphs in these classes.

Keywords

Adjacent distinguishing edge coloring; Split graph; Indifference graph

1. INTRODUCTION

In this paper, we consider simple, finite and undirected graphs. We denote a graph G with vertex set $V(G)$ and edge set $E(G)$ by $G = (V(G), E(G))$, and the maximum degree of G by $\Delta(G)$. A *proper edge coloring* of G is an assigment of colors to the edges of G so that no adjacent edges receive the same color. The minimum number of colors for which a graph G has a proper edge coloring is the *chromatic index* of G, denoted by $\chi'(G)$. Given a proper edge coloring of G, the *set of colors* of a vertex $v \in V(G)$ is the set of colors assigned to the edges incident to v, denoted by $C(v)$. Two vertices u and v are *distinguishable* when $C(u) \neq C(v)$. An *adjacent vertex distinguishing* (AVD) edge coloring of a graph G is a proper edge coloring of G such that every two adjacent vertices are distinguishable. Note that there is no AVD edge coloring for the complete graph K_2. The *AVD Edge Coloring Problem* was introduced by Zhang et al. in 2002 [8] and consists in determing the least number of colors for an AVD edge coloring of a graph G, called *AVD chromatic index* and denoted $\chi'_a(G)$. In the same paper, Zhang et al. [8] mentioned, without giving more details, that some network problems can be converted to the AVD Edge Coloring Problem. They also presented the first results on the AVD chromatic index.

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THEOREM 1. *[8] If G is the disjoint union of n connected components $G_1, G_2, \ldots, G_n$, and $|V(G_i)| \geq 3$, $1 \leq i \leq n$, then $\chi'_a(G) = \max\{\chi'_a(G_i), 1 \leq i \leq n\}$.*

By Theorem 1, it is sufficient to consider connected graphs to solve the AVD Edge Coloring Problem. Hence, from now, we consider all the graphs connected. Zhang et al. [8] also determined the AVD chromatic index for trees, complete graphs, cicles, and complete bipartite graphs. In the same paper, they proposed the following conjecture.

CONJECTURE 2. *[8] If G is a connected graph and $|V(G)| \geq 6$, $\chi'_a(G) \leq \Delta(G) + 2$.*

Balister et al. [1] proved the Conjecture 2 for bipartite graphs and for every graph G with $\Delta(G) = 3$.

Once the AVD chromatic index were determined for complete graphs and some results were known for bipartite graphs, it is interesting to consider this problem on split graphs, since every split graph is an edge disjoint union of a complete graph and a bipartite graph. Precisely, a *split graph* $G = [Q, S]$ is a graph whose vertex set can be partitioned into a clique Q and an independent set S, where a *clique* is a set of pairwise adjacent vertices and an *independent set* is a set of pairwise non-adjacent vertices. A *complete split graph* is a split graph $G = [Q, S]$ where every vertex $u \in Q$ is adjacent to every vertex $v \in S$. In [7], Vilas-Bôas and Mello proved that the Conjecture 2 is true for complete split graphs. Let $G = [Q, S]$ be a complete split graph. They also show that if G has odd $\Delta(G)$ and $|Q| > |S|^2$, then $\chi'_a(G) = \Delta(G) + 2$. If G has $|Q| \geq 2$, they proved that $\chi'_a(G) = \Delta(G) + 1$ when $\Delta(G)$ is even or when $|Q| \leq |S|^2 - |S|$. Note that when $|Q| = 1$ the complete split graph is the complete bipartite graph $K_{1,m}$ for which $\chi'_a(G)$ was determined by Zhang et al. [8]. Therefore, it remains to determine $\chi'_a(G)$ when $|Q| \geq 2$, $\Delta(G)$ is odd, and $|S|^2 - |S| < |Q| \leq |S|^2$. In this case, Vilas-Bôas and Mello [7] conjecture that $\chi'_a(G) = \Delta(G) + 1$.

In the same paper, Vilas-Bôas and Mello [7] considered the split-indifference graphs. An *indifference graph* is a graph whose vertex set can be linearly ordered such that vertices belonging to the same clique are consecutive. An *split-indifference graph* is a graph that is simultaneously split and indifference. They proved that the Conjecture 2 is true for split-indifference graphs. Furthermore, for a split- indifference graph G, they determined $\chi'_a(G)$ when $|V(G)|$ is even and in some cases when $|V(G)|$ is odd.

In this paper we conclude the work of Vilas-Bôas and Mello [7], presenting the AVD chromatic index for the remaining complete split graphs and split-indifference graphs.

2. THEORETICAL FRAMEWORK

In this section, we present definitions and previous results that are important to the development of this work.

If a graph G satisfies $|E(G)| > \Delta(G)\left\lfloor\frac{V(G)}{2}\right\rfloor$, then G is *overfull*. If G has a subgraph H with $\Delta(H) = \Delta(G)$ and H is overfull, then G is *subgraph-overfull*. The overfull and subgraph-overfull graphs have $\chi'(G) = \Delta(G) + 1$.

A *universal vertex* of a graph G is a vertex of G with degree $|V(G)|-1$. If G has a universal vertex and the number of edges in the complement of G is less than $\frac{\Delta(G)}{2}$, then G is overfull. In 1981, Plantholt determined the chromatic index for graphs with universal vertex, as presented in the next theorem.

Theorem 3. *[5] Let G be a graph with universal vertex. $\chi'(G) = \Delta(G)$ if, and only if, G is not overfull.*

Given a graph G with a set of vertices $X \subseteq V(G)$, the *subgraph of G induced by X* is the subgraph $H = (V(H), E(H))$ with $V(H) = X$ and $E(H) = \{uv : u, v \in X \wedge uv \in E(G)\}$. The *core* of a graph G, denoted $\Lambda(G)$, is the set of maximum degree vertices of G. Fournier [3] determined the chromatic indices of graphs with acyclic $G[\Lambda(G)]$.

Theorem 4. *[3] If $G[\Lambda(G)]$ is a forest, then $\chi'(G) = \Delta(G)$.*

When a graph G does not have maximum degree adjacent vertices, its core is an independent set and consequently $G[\Lambda(G)]$ is a forest. By Theorem 4, $\chi'(G) = \Delta(G)$. Consider an edge coloring of G with $\Delta(G)$ colors. If any two vertices of G have distinct degrees, the cardinalities of its sets of colors are different. Hence, any two sets of colors are distinguishable. Therefore, $\chi'_a(G) = \Delta(G)$. This result is presented in the following theorem of Zhang et al.

Theorem 5. *[8] If G is a graph where the degree of every two adjacent vertices are different, then $\chi'_a(G) = \Delta(G)$.*

On the other hand, if G is a graph with two adjacent maximum degree vertices u and v, then $C(u) = C(v)$ for any edge coloring of G with $\Delta(G)$ colors. This observation implies a lower bound for the AVD chromatic index of G, as follows.

Theorem 6. *[8] If G is a graph with two adjacent maximum degree vertices, then $\chi'_a(G) \geq \Delta(G) + 1$.*

The lower bound given by Theorem 6 is tight as the following two theorems show.

Theorem 7. *[8] If G is a tree with $|V(G)| \geq 3$, then*

$$\chi'_a(G) = \begin{cases} \Delta(G), & \text{if there are no maximum degree adjacent vertices;} \\ \Delta(G) + 1, & \text{otherwise.} \end{cases}$$

Theorem 8. *[8] If K_n is a complete graph with n vertices, then*

$$\chi'_a(K_n) = \begin{cases} \Delta(K_n) + 1, & \text{if } n \text{ is odd;} \\ \Delta(K_n) + 2, & \text{if } n \text{ is even.} \end{cases}$$

Theorem 9. *[8] If $K_{m,n}$ is a complete bipartite with $1 \leq m \leq n$, then*

$$\chi'_a(K_{m,n}) = \begin{cases} n, & \text{if } m < n; \\ n + 2, & \text{if } m = n > 1. \end{cases}$$

A *proper total coloring* of a graph $G = (V(G), E(G))$ is an assignment of colors to $V(G) \cup E(G)$ such that no adjacent elements have the same color. The least number of colors that allows a proper total coloring of a graph G is the *total chromatic number*, denoted as $\chi''(G)$. Chen et al. [2] determine the total chromatic number for complete split graphs, presented in the next theorem.

Theorem 10. *[2] Let G be a split graph. If $\Delta(G)$ is even, then $\chi''(G) = \Delta(G) + 1$.*

Considering the complete split graphs, the known results on the AVD chromatic index are presented below.

Theorem 11. *[7] If $G = [Q, S]$ is a complete split graph and $\Delta(G)$ is odd, then $\chi'_a(G) \leq \Delta(G) + 2$.*

Theorem 12. *[7] Let $G = [Q, S]$ be a complete split graph where $|Q| \geq 2$. If $\Delta(G)$ is even or $|Q| \leq |S|^2 - |S|$, then $\chi'_a(G) = \Delta(G) + 1$. If $\Delta(G)$ is odd and $|Q| > |S|^2$, then $\chi'_a(G) = \Delta(G) + 2$.*

Therefore, by Theorem 12, it remains to determine $\chi'_a(G)$ when $\Delta(G)$ is odd and $|S|^2 - |S| < |Q| \leq |S|^2$. For this case, Vilas-Bôas and Mello presented the folowing conjecture.

Conjecture 13. *[7] If $G = [Q, S]$ is a complete split graph, $\Delta(G)$ is odd, and $|S|^2 - |S| < |Q| \leq |S|^2$, then $\chi'_a(G) = \Delta(G) + 1$.*

Theorem 14 shows a characterization for split-indifference graphs, which partition its vertex set into cliques. This partition was used by Vilas-Bôas and Mello [7] on their results.

Theorem 14. *[4] A graph G is an split-indifference graph if and only if*

1. *G is a complete graph, or*
2. *G is the union of two complete graphs G_1, G_2, such that $G_1 \setminus G_2 = K_1$, or*
3. *G is the union of three complete graphs G_1, G_2, and G_3, such that $G_1 \setminus G_2 = K_1$, $G_3 \setminus G_2 = K_1$, $V(G_1) \cap V(G_2) \neq \emptyset$, and $V(G_1) \cup V(G_3) = V(G)$, or*

4. *G is the union of three complete graphs G_1, G_2, G_3, such that $G_1 \setminus G_2 = K_1$, $G_3 \setminus G_2 = K_1$, and $V(G_1) \cap V(G_3) = \emptyset$.*

The Figure 2 shows a schematic representation of the split-indifference graphs according to the characterization given by Theorem 14, where each circle represents a clique with its respective vertices.

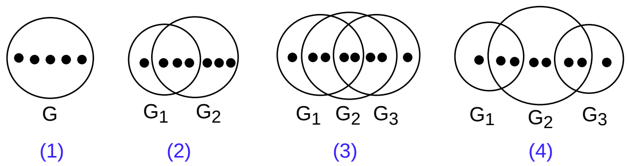

Figure 1: The four cases of the characterization of split-indifference graphs according to Theorem 14.

Let G be a split-indifference graph. If G satisfies the Case (1) of Theorem 14, the AVD chromatic index is determined by Theorem 8. Consider that G satisfies the Case (2) of Theorem 14. If $|\Lambda(G)| = 1$, then G is a tree and the AVD chromatic index is determined by Theorem 7. If $|\Lambda(G)| \geq 2$, then $\chi'_a(G)$ is determined by Vilas-Bôas and Mello, as follows.

Theorem 15. *[6] Let G be a split-indifference graph such that G is the union of two complete graphs G_1 and G_2, and $G_1 \setminus G_2 = K_1$. If $|V(G)|$ is odd and $|\Lambda(G)| \geq 2$, then $\chi'_a(G) = \Delta(G) + 1$.*

Theorem 16. *[6] Let G be a split-indifference graph such that G is the union of two complete graphs G_1 and G_2, and $G_1 \setminus G_2 = K_1$. If $|V(G)|$ is even and $|\Lambda(G)| \geq 2$, then*

$$\chi'_a(G) = \begin{cases} \Delta(G)+1, & \text{if } 2 \leq |\Lambda(G)| \leq \frac{3\Delta(G)+1}{4}; \\ \Delta(G)+2, & \text{if } |\Lambda(G)| > \frac{3\Delta(G)+1}{4}. \end{cases}$$

Consider that G satisfies Case (3) of Theorem 14. Suppose without loss of generality that $|V(G_1) \cap V(G_2)| \geq |V(G_2) \cap V(G_3)|$. There are two cases: when $|V(G)|$ is odd and when $|V(G)|$ is even.

Theorem 17. *[6] Let G be a split-indifference graph such that G is the union of three complete graphs G_1, G_2, and G_3, $G_1 \setminus G_2 = K_1$, $G_3 \setminus G_2 = K_1$, $V(G_1) \cap V(G_2) \neq \emptyset$, and $V(G_1) \cup V(G_3) = V(G)$. If $|V(G)|$ is odd, then*

$$\chi'_a(G) = \begin{cases} \Delta(G), \text{ if } \Lambda(G) = 1 \text{ and} \\ \qquad |V(G_1) \cap V(G_2)| = |V(G_2) \cap V(G_3)|; \\ \Delta(G), \text{ if } \Lambda(G) = 1 \text{ and} \\ \qquad |V(G_1) \cap V(G_2)| = |V(G_2) \cap V(G_3)| + 2; \\ \Delta(G)+1, \text{ otherwise.} \end{cases}$$

Theorem 18. *[6] Let G be a split-indifference graph such that G is the union of three complete graphs G_1, G_2, and G_3, $G_1 \setminus G_2 = K_1$, $G_3 \setminus G_2 = K_1$, $V(G_1) \cap V(G_2) \neq \emptyset$, and $V(G_1) \cup V(G_3) = V(G)$. If $|V(G)|$ is even, then*

$$\chi'_a(G) = \begin{cases} \Delta(G), & \text{if } \Lambda(G) = 1; \\ \Delta(G)+1, & \text{if } \Lambda(G) > \frac{3\Delta(G)+1}{4}. \text{ and }; \\ \Delta(G)+2, & \text{otherwise.} \end{cases}$$

Finally, consider that G satisfies Case (4) of Theorem 14. There are two cases: when $|V(G)|$ is odd and when $|V(G)|$ is even.

Theorem 19. *[6] Let G be a split-indifference graph such that G is the union of three complete graphs G_1, G_2, G_3, where $G_1 \setminus G_2 = K_1$, $G_3 \setminus G_2 = K_1$, and $V(G_1) \cap V(G_3) = \emptyset$. If $|V(G)|$ is even, then $\chi'_a(G) = \Delta(G) + 1$, otherwise, $\chi'_a(G) \leq \Delta(G) + 2$.*

When $|V(G)|$ is odd, Vilas-Bôas and Mello [6] presented the following partial results.

Theorem 20. *[6] Let G be a split-indifference graph such that G is the union of three complete graphs G_1, G_2, G_3, where $G_1 \setminus G_2 = K_1$, $G_3 \setminus G_2 = K_1$, and $V(G_1) \cap V(G_3) = \emptyset$. If $|V(G)|$ is odd then*

$$\chi'_a(G) = \begin{cases} \Delta(G)+1, & \text{if } |V(G_1) \cap V(G_2)| \leq \frac{\Delta(G)+1}{2}; \\ \Delta(G)+2, & \text{if } |V(G_1) \cap V(G_2)| \geq \frac{3\Delta(G)+1}{4}. \end{cases}$$

Now, it remains to consider the Case (4) when $|V(G)|$ is odd and $\frac{\Delta(G)+1}{2} < |V(G_1) \cap V(G_2)| < \frac{3\Delta(G)+1}{4}$.

Theorem 21. *[6] Let G be a split-indifference graph such that G is the union of three complete graphs G_1, G_2, G_3, where $G_1 \setminus G_2 = K_1$, $G_3 \setminus G_2 = K_1$, and $V(G_1) \cap V(G_3) = \emptyset$. If $|V(G)|$ is odd, $|V(G_1) \cap V(G_2)| = \frac{\Delta(G)+1}{2} + p$, $|V(G_2) \setminus (V(G_1) \cup V(G_3))| \geq p$, where p is an integer, $0 \leq p < \frac{\Delta(G)-1}{4}$, and $|V(G_1) \cap V(G_2)| \geq |V(G_2) \cap V(G_3)|$, then $\chi'_a(G) = \Delta(G) + 1$.*

For the other cases, Vilas-Bôas and Mello posed Conjecture 22.

Conjecture 22. *[6] Let G be a split-indifference graph such that G is the union of three complete graphs G_1, G_2, G_3, where $G_1 \setminus G_2 = K_1$, $G_3 \setminus G_2 = K_1$, and $V(G_1) \cap V(G_3) = \emptyset$. If $|V(G)|$ is odd, $|V(G_1) \cap V(G_2)| = \frac{\Delta(G)+1}{2} + p$, $|V(G_2) \setminus (V(G_1) \cup V(G_3))| < p$, where p is an integer, $0 \leq p < \frac{\Delta(G)-1}{4}$, and $|V(G_1) \cap V(G_2)| \geq |V(G_2) \cap V(G_3)|$, then $\chi'_a(G) = \Delta(G) + 2$.*

3. RESULTS

In this section we present the AVD chromatic index for the split graphs that satisfies the hypothesis of conjectures 13 and 22.

To investigate the Conjecture 13, we consider a complete split graph $G = [Q, S]$ with odd $\Delta(G)$ and $|S^2| - |S| < |Q| \leq |S^2|$. Then we construct a new graph G^* by adding a new vertex v^* to G and connect v^* to every vertex of Q. Since G^* has a universal vertex, $\chi'(G^*) = \Delta(G^*)$ if and only if G^* is not overfull, by Theorem 3. The next lemma proves that when G satisfies the hypothesis of the Conjecture 13, the graph G^* is not overfull.

Lemma 23. *Let $G = [Q, S]$ be a complete split graph with odd $\Delta(G)$ and $|Q| \leq |S^2|$. If G^* is the graph obtained from G by adding a new vertex v^* adjacent to every vertex of Q, then G^* is not overfull.*

Proof. Let $G = [Q, S]$ be a complete split graph with odd $\Delta(G)$ and $|Q| \leq |S^2|$. Construct a graph G^* from G, adding a new vertex v^* and connecting v^* to every vertex of Q. Thus, $|E(G^*)| = |E(G)|+|Q| = \frac{|Q|(|Q|-1)}{2}+|Q||S|+|Q| = |Q|\frac{|Q|+|S|}{2} + |Q|\frac{|S|}{2} + \frac{|Q|}{2}$. By hypothesis $|Q| \leq |S|^2$, then $|Q|\frac{|Q|+|S|}{2} + |Q|\frac{|S|}{2} + \frac{|Q|}{2} \leq |Q|\frac{|Q|+|S|}{2} + |Q|\frac{|S|}{2} + \frac{|S|^2}{2} = |Q|\frac{|Q|+|S|}{2} + |S|\frac{|Q|+|S|}{2}$. By hypothesis, $\Delta(G) = |Q|+|S|-1$ is odd, so $|Q|+|S|$ is even. Then, $|E(G^*)| \leq |Q|\frac{|Q|+|S|}{2} + |S|\frac{|Q|+|S|}{2} = (|Q|+|S|)\lfloor\frac{|Q|+|S|+1}{2}\rfloor = \Delta(G^*)\lfloor\frac{|V(G^*)|}{2}\rfloor$.

Therefore, G^* is not overfull. □

The next theorem shows Conjecture 13 is true.

Theorem 24. *If $G = [Q, S]$ is a complete split graph, $\Delta(G)$ is odd, and $|S^2| - |S| < |Q| \leq |S^2|$, then $\chi'_a(G) = \Delta(G) + 1$.*

Proof. Let $G = [Q, S]$ be a complete split graph, with odd $\Delta(G)$ and $|S^2| - |S| < |Q| \leq |S^2|$. By the hypothesis, $|S| \geq 2$ and $|Q| \geq 4$. So, G has maximum degree adjacent vertices and $\chi'_a(G) \geq \Delta(G) + 1$ by Theorem 6.

We create a new graph G^* by adding a new vertex v^* to G and connect v^* to every vertex of Q. Since G^* has a universal vertex, $\chi'(G^*) = \Delta(G^*)$ if and only if G^* is not overfull, by Theorem 3. By Lemma 23, G^* is not overfull. Thus, consider an edge-coloring of G^* with $\Delta(G^*)$ colors and remove the vertex v^*.

The resulting graph is G with an AVD-edge-coloring. In fact, the degree of each vertex in S is $|Q|$ and the degree of each vertex in Q is $|Q| - 1 + |S|$. Since $|S| \geq 2$, we have $|Q| < |Q| - 1 + |S|$. Then, for any pair of vertices $v \in Q$ and $u \in S$, $|C(v)| > |C(u)|$ and therefore $C(v) \neq C(u)$. Moreover, there is exactly one color missing in each vertex of Q. By construction, the colors missing in any two vertices of Q are pairwise distinct, since this colors were used to color edges incident to v^*. Hence, the sets of colors of any two vertices in Q are different. Therefore, it is an AVD-edge-coloring for G and $\chi'_a(G) = \Delta(G^*) = \Delta(G) + 1$. □

Now, we prove that Conjecture 22 is true. Let G be a split-indifference graph such that G is the union of three complete graphs G_1, G_2, G_3, where $G_1 \setminus G_2 = K_1$, $G_3 \setminus G_2 = K_1$, and $V(G_1) \cap V(G_3) = \emptyset$.

Theorem 25. *Let G be a split-indifference graph without universal vertex. If $|V(G)|$ is odd, $|V(G_1) \cap V(G_2)| = (\Delta(G)+1)/2 + p$, $|V(G_2) \setminus (V(G_1) \cup V(G_3))| < p$, $p \in \mathbb{Z}$, $0 \leq p < \frac{\Delta(G)-1}{4}$ and $|V(G_1) \cap V(G_2)| \geq |V(G_2) \cap V(G_3)|$, then $\chi'_a(G) = \Delta(G) + 2$.*

Proof. Construct a graph G^* by adding a vertex v^* adjacent to every maximum degree vertex of G. Observe that $\Delta(G) = |Q|$, $\Delta(G^*) = |Q| + 1$, and there is no universal vertex in G^*.

Consider the induced subgraph $H = G^*[V(G_1) \cup V(G_2) \cup \{v^*\}]$. Note that $\Delta(H) = \Delta(G^*)$ and H has a universal vertex. Now we will show that H is overfull and therefore, G^* is subgraph-overfull.

Note that the number of edges in the complement of H is $2|V(G_2) \setminus (V(G_1) \cup V(G_3))| + |V(G_2) \cap V(G_3)| + 1$. Since $|V(G_2) \cap V(G_3)| = |Q| - |V(G_2) \setminus (V(G_1) \cup V(G_3))| - |V(G_1) \cap V(G_2)|$, the number of edges in the complement of H is $|V(G_2) \setminus (V(G_1) \cup V(G_3))| + |Q| - |V(G_1) \cap V(G_2)| + 1$. By hypothesis, $|V(G_1) \cap V(G_2)| = (\Delta(G)+1)/2 + p$. Hence, the number of edges in the complement of H is $|V(G_2) \setminus (V(G_1) \cup V(G_3))| + |Q| - (\Delta(G)+1)/2 - p + 1 = |V(G_2) \setminus (V(G_1) \cup V(G_3))| + |Q| - (|Q|+1)/2 - p + 1 = \frac{\Delta(H)}{2} + |V(G_2) \setminus (V(G_1) \cup V(G_3))| - p$. By hypothesis, $|V(G_2) \setminus (V(G_1) \cup V(G_3))| < p$. Thus, the number of edges in the complement of H is less than $\frac{\Delta(H)}{2}$, which implies that H is overfull and consequently G^* is subgraph-overfull. So, $\chi'(G^*) = \Delta(G^*) + 1$.

Therefore, there is no edge coloring for G with $\Delta(G^*) = \Delta(G) + 1$ colors that allows pairwise distinct sets of colors for the maximum degree vertices. So, $\chi'_a(G) \geq \Delta(G^*) + 1 = \Delta(G) + 2$. By Theorem 11, $\chi'_a(G) = \Delta(G) + 2$. □

4. CONCLUSION

Considering a complete split graph G, we conclude that if $V(G)$ has a partition into a clique Q and an independent set S such that $|Q| > |S|^2$ and $\Delta(G)$ is odd, then $\chi'_a(G) = \Delta(G) + 2$, otherwise, $\chi'_a(G) = \Delta(G) + 1$.

For split indifference graphs, four cases were considered, according to the characterization of these graphs given by Theorem 14. Before considering such cases it is importante to note that if $|\Lambda(G)| = 1$, then G is a tree and the AVD chromatic index is determined by Theorem 7. Then, the following conclusions are about graphs with $|\Lambda(G)| \geq 2$. When G is a complete graph, the AVD Edge Coloring Problem is solved by Theorem 8. For the next cases, consider G_1, G_2 and G_3 as described in Theorem 14. When G is the union of two complete graphs G_1 and G_2, the AVD chromatic index is determined by Theorems 15 and 16. When G is the union of three complete graphs, G_1, G_2, and G_3, such that $V(G_1) \cap V(G_3) \neq \emptyset$, the AVD-Edge Coloring Problem is solved by Theorems 17 and 18. Finally, if G is the union of three complete graphs, G_1, G_2, and G_3, such that $V(G_1) \cap V(G_3) = \emptyset$, then the AVD chromatic index is determined by Theorems 19, 20, 21 and 25.

As a future work, we plan to investigate the AVD chromatic indices of other split graphs, such as the split-comparability graphs, a superclass of split-indifference graphs.

5. REFERENCES

[1] P. N. Balister, E. Gyori, J. Lehel, and R. H. Schelp. Adjacent vertex distinguishing edge-colorings. *SIAM Journal on Discrete Mathematics*, 21(1):237–250, 2007.

[2] B.-L. Chen, H.-L. Fu, M. Ko, et al. Total chromatic number and chromatic index of split graphs. *Journal of Combinatorial Mathematics and Combinatorial Computing*, 17, 1995.

[3] J.-C. Fournier. Colorations des arétes d'un graphe. *Cahiers du CERO*, 15:311–314, 1973.

[4] C. Ortiz, N. Maculan, and J. L.Szwarcfiter. Characterizing and edge-coloring split-indiference graphs. *Discrete applied mathematics*, 82:209–217, 1998.

[5] M. Plantholt. The chromatic index of graphs with a spanning star. *Journal of graph theory*, 5(1):45–53, 1981.

[6] A. d. M. Vilas-Bôas. Coloração de arestas semiforte de grafos split. Master's thesis, Universidade Estadual de Campinas, Instituto de Computação, Campinas, SP, 2015.

[7] A. d. M. Vilas-Bôas and C. P. de Mello. The avd-edge-coloring conjecture for some split graphs. *Matemática contemporânea*, 44:1–10, 2015.

[8] Z. Zhang, L. Liu, and J. Wang. Adjacent strong edge coloring of graphs. *Applied mathematics letters*, 15(5):623–626, 2002.