On a conjecture on edge-colouring join graphs

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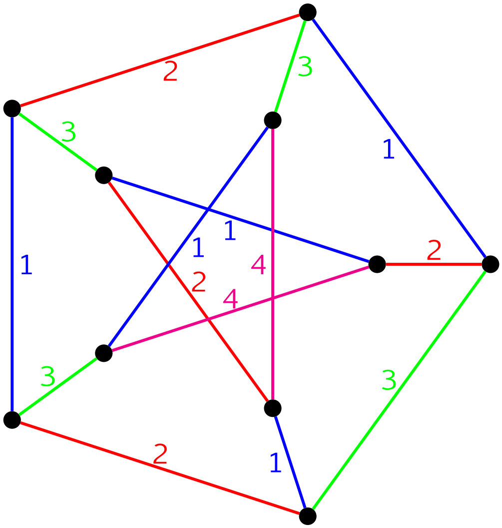
WPCCG'17

1. Introduction

2. Contribution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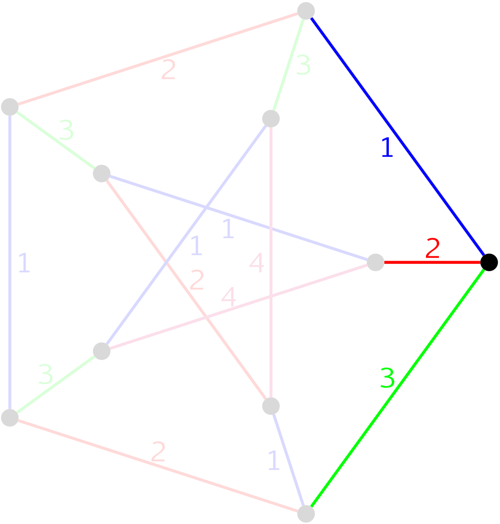
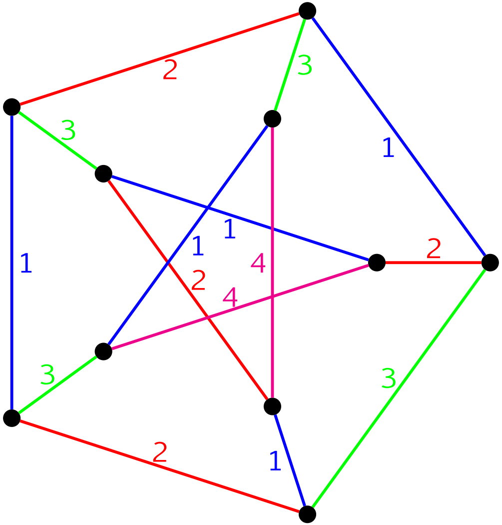


$$\chi'(G) \geqslant \Delta(G)$$



$$\chi'(G) \geqslant \Delta(G)$$

Vizing's Theorem (1964)

For any graph G, $\chi'(G) \leqslant \Delta(G) + 1$.

$$\begin{array}{ll} \chi'(G) = \Delta(G) & \textit{Class 1} \\ \chi'(G) = \Delta(G) + 1 & \textit{Class 2} \end{array}$$

- Deciding if a graph is *Class 1* is $\mathcal{NP}$-complete(Holyer, 1981)

V. G. Vizing (1964). *On an estimate of the chromatic class of a p-graph*. Diskret. Analiz., 3:25–30, in Russian

I. Holyer (1981). *The $\mathcal{NP}$-completeness of edge-colouring*. SIAM J. Comput., 10(4):718–720

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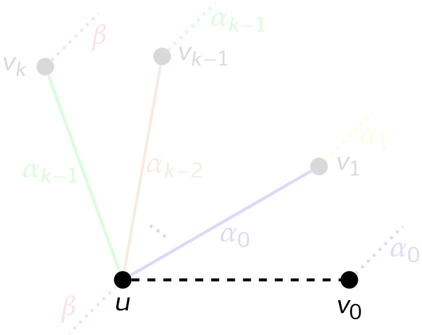
Vizing's recolouring procedure



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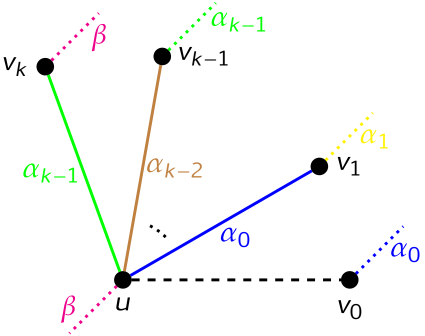

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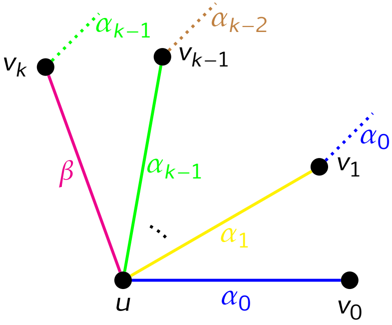

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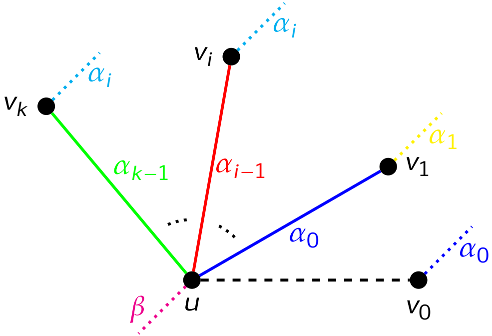
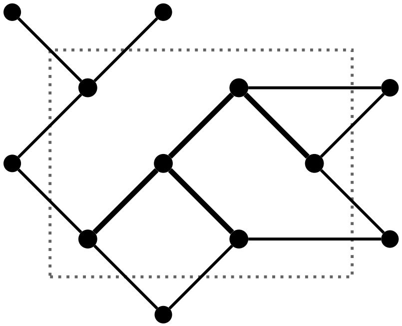
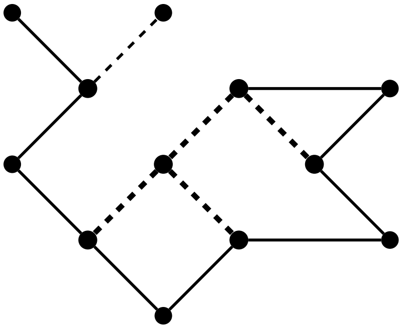


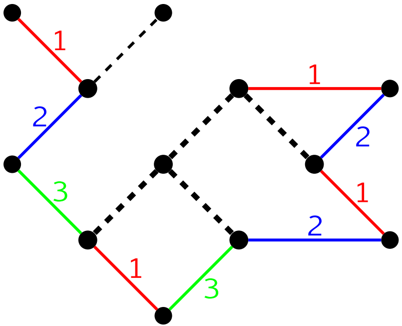
Theorem (Fournier, 1977)

Every graph with acyclic core is *Class 1*.



J.-C. Fournier (1977). *Méthode et théorème générale de coloration des arêtes.* J. Math. Pures Appl., 56:437–453




1
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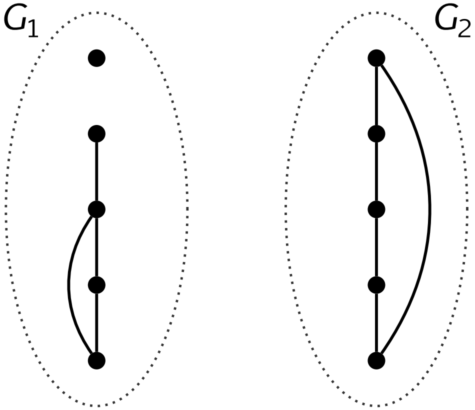
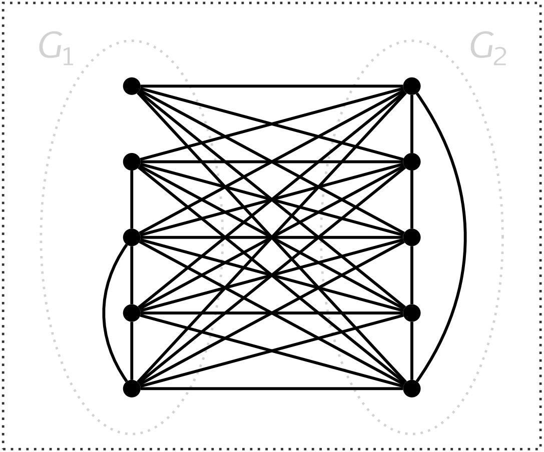
Join graphs

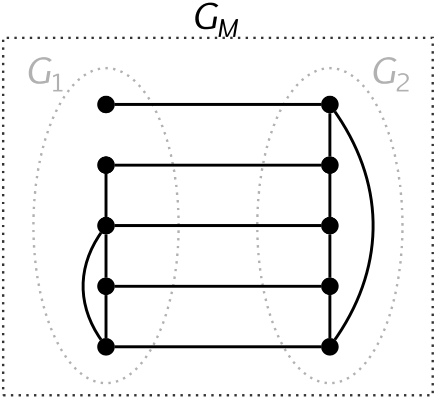


Join graphs

$$G = G_1 * G_2$$



Join graphs with $k := n_1 = n_2$ and $d := \Delta_1 = \Delta_2$



$$\Delta(G_M) = d + 1$$

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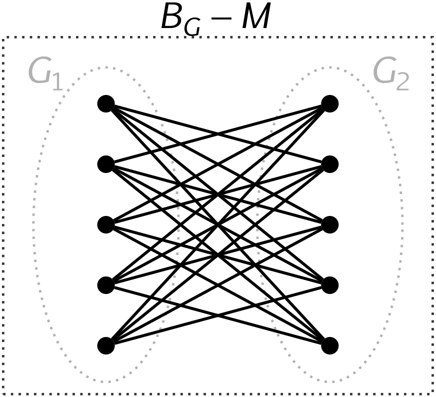
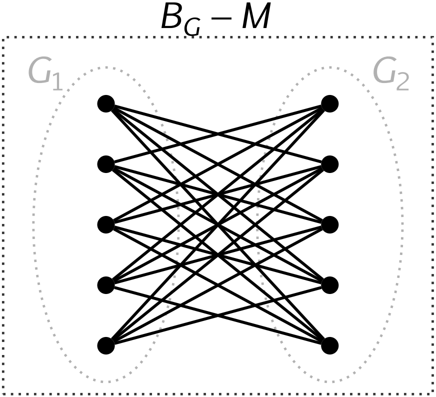


$$\Delta(B_G - M) = k - 1 = \chi'(B_G - M)$$
$$\therefore \chi'(G_M) = d + 1 \Longrightarrow \chi'(G) = k + d$$

Join graphs with $k := n_1 = n_2$ and $d := \Delta_1 = \Delta_2$



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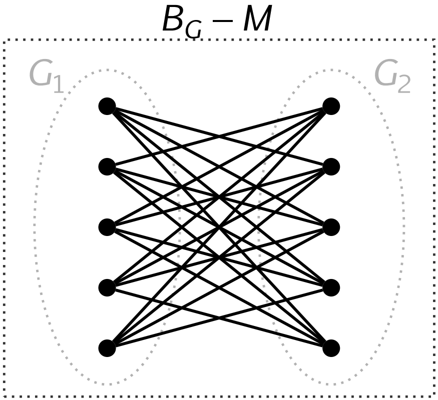

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Conjecture (Zorzi e Zatesko, 2016; Lima et. al. , 2015)

If $n_1 = n_2$ and $\Delta_1 = \Delta_2$, then G is *Class 1*.

Theorem (Zorzi e Zatesko, 2016)

If $n_1 = n_2, \Delta_1 = \Delta_2$, and

$$|V(G_2) \setminus V(\Lambda[G_2])| \geqslant |\{u \in V(\Lambda[G_1]) : d_{\Lambda[G_1]}(u) > 1\}| + |\{C \text{ connected component of } \Lambda[G_1] : |V(C)| = 2\}|,$$

then G is *Class 1*.

A. R. C. Lima, G. Garcia, L. M. Zatesko, and S. M. de Almeida (2015). *On the chromatic index of cographs and join graphs*. Electron. Notes Discrete Math., 50:433–438

A. Zorzi and L. M. Zatesko (2016). *On the chromatic index of join graphs and triangle-free graphs with large maximum degree*. Discrete Appl. Math., article in press,

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1. Introduction

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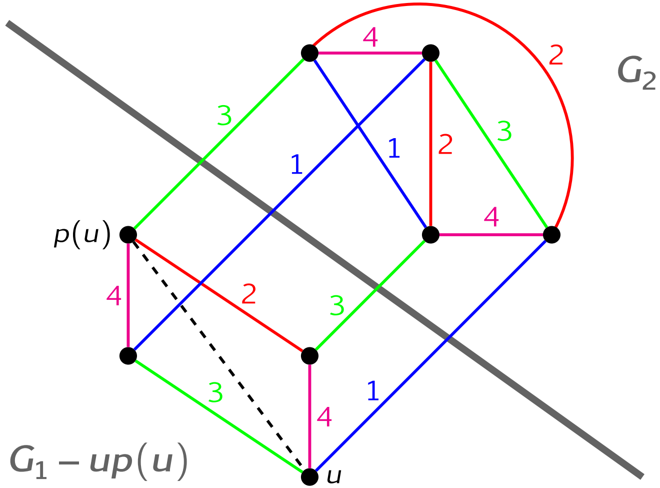
Proposition

If a graph G has maximum degree $\Delta > 1$ and an acyclic core with s vertices, then G has at least

$$\max\left\{\Delta - 1, s - \left\lfloor \frac{s-2}{\Delta - 1} \right\rfloor\right\}$$

vertices of degree less than Δ.


4
2
G_2
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$p(u)$
4
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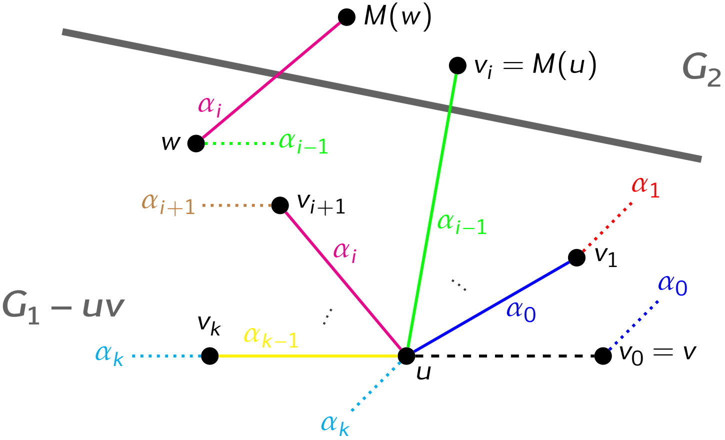
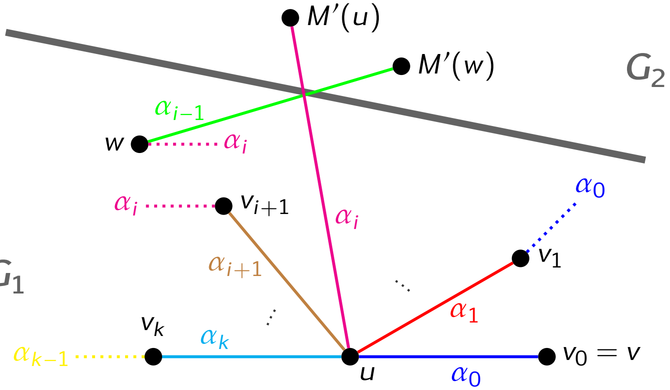
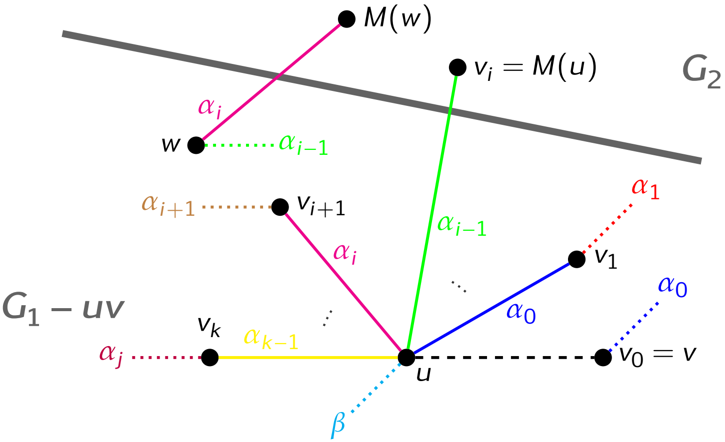
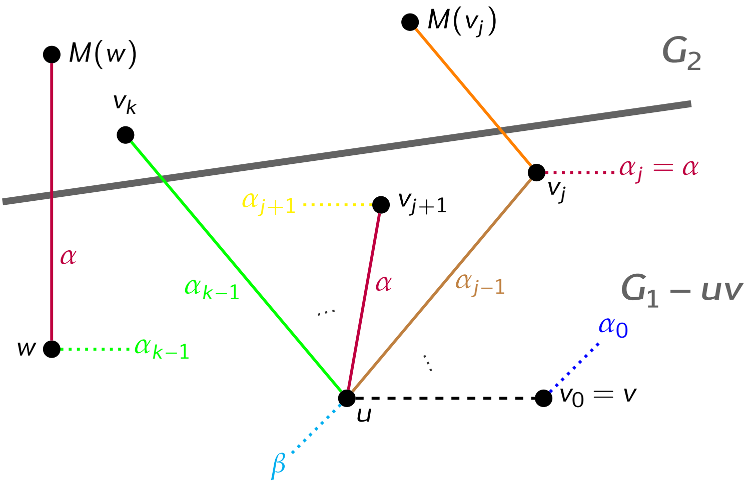
$G_1 - up(u)$
u

$$\sum_{v \in V(G_1)} ((d+1) - d_H(v)) \geqslant d+1.$$


$M(w)$
$v_i = M(u)$
G_2
α_i
w
α_{i-1}
α_{i+1}
v_{i+1}
α_1
α_{i-1}
α_i
v_1
α_0
$G_1 - uv$
v_k
α_{k-1}
α_0
α_k
$v_0 = v$
u
α_k


$M'(u)$
$M'(w)$
G_2
α_{i-1}
w
α_i
α_0
α_i
v_{i+1}
α_i
v_1
α_{i+1}
G_1
α_1
v_k
α_k
α_{k-1}
$v_0 = v$
u
α_0


$M(w)$
$v_i = M(u)$
G_2
α_i
w
α_{i-1}
α_{i+1}
v_{i+1}
α_1
α_{i-1}
α_i
v_1
α_0
$G_1 - uv$
α_0
v_k
α_{k-1}
α_j
$v_0 = v$
u
β


$M(v_j)$
$M(w)$
G_2
v_k
$\alpha_j = \alpha$
v_j
α_{j+1}
v_{j+1}
α
α_{k-1}
α
α_{j-1}
$G_1 - uv$
α_0
w
α_{k-1}
$v_0 = v$
u
β

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